Deskew Notes

Automatic cropping of rotated images

Approach

- Determine the formulae for the corners of the rotated image
- Determine the equations for each of the sides of the rotated image
- Determine the points where the sides of the rotated image intercept the sides of the frame (un-rotated image)
- Crop the rotated image to be within horizontal and vertical lines drawn through these points, with the lines chosen to maximize the cropped image.

Required algebraic steps

In order to discover and check the methodology, this document presents almost all possible formulas and equations pertaining to cropping a rotted image; however, due to symmetry, far fewer are aactually needed.

Those required are:

Inputs	Outputs
width and height of image before rotation angle of rotation	coordinates of ends of left side of rotated image
coordinates of ends of left side of rotated image	equation for left side of rotated image
equation for left side of rotated image equation for bottom of un-rotated image (frame)	x intercept of bottom of frame and left side of rotated image, piBR

width and height of image before rotation angle of rotation	coordinates of ends of bottom of rotated image
coordinates of ends of bottom of rotated image	equation for bottom of rotated image
equation for bottom of rotated image equation for right side of frame	y intercept of right side of frame and bottom of rotated image, piRB

The above is for anticlockwise rotation of the image (positive rotation angle). If rotation is anticlockwise, rotation angle = – rotation angle.

This method is intended for rotation angles up to about 10 degrees.

It is written for **landscape** images. For **portrait**, due to symmetry the same formulae will work.

The needed derivations appear in the body of the document. One that were not used are in the appendix.

Formulae for the corners of the rotated image

The formulae for rotating an image anticlockwise about the origin are:

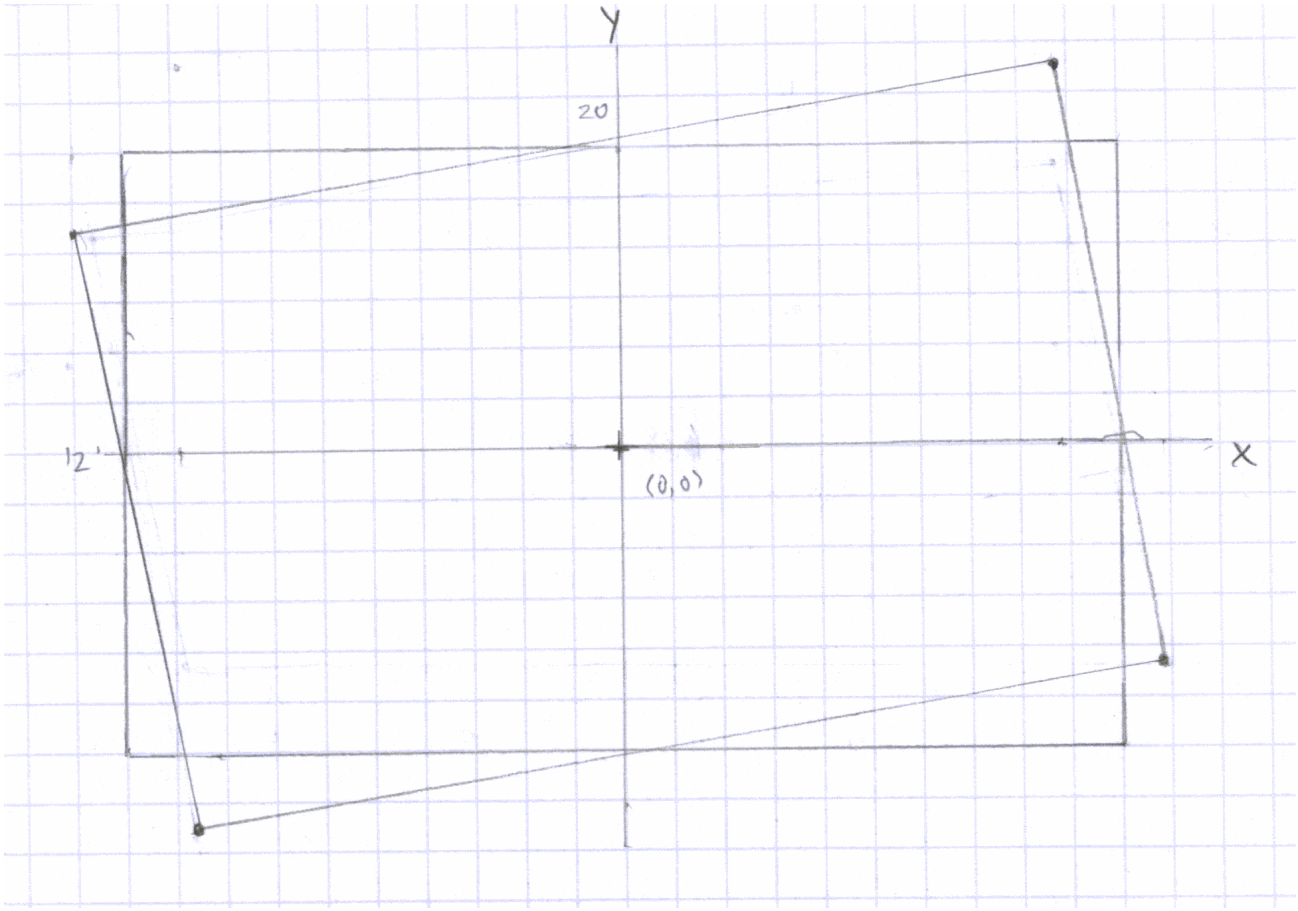
$x' = x \cos \alpha - y \sin \alpha$
$y' = x \sin \alpha + y \cos \alpha$

With the centre of the images being the origin and the centre of rotation, the coordinates of the corners are:

Corner		**Un-rotated**	**Rotated**	
			Formula	**Calculated***
TL	x	–w/2	$-w/2 \cos \alpha - h/2 \sin \alpha$	– 10.9
	y	h/2	$-w/2 \sin \alpha + h/2 \cos \alpha$	4.2
TR	x	w/2	$w/2 \cos \alpha - h/2 \sin \alpha$	8.8
	y	h/2	$w/2 \sin \alpha + h/2 \cos \alpha$	7.6
BL	x	– w/2	$-w/2 \cos \alpha + h/2 \sin \alpha$	– 8.8
	y	– h/2	$-w/2 \sin \alpha - h/2 \cos \alpha$	– 7.6
BR	x	w/2	$w/2 \cos \alpha + h/2 \sin \alpha$	10.9
	y	– h/2	$w/2 \sin \alpha - h/2 \cos \alpha$	4.2
* For w = 20, h = 12, $\alpha = 10°$ (plotted below)				

The result from Irfanview for a positive rotation angle has the rotated image clockwise; the top left corner is higher than the top right.

But plotting shows that **these formulas rotate the image anticlockwise** for positive α.



Determine the equations for each of the sides of the rotated image

Left rotated side

The BL corner is at (–w/2 cos α + h/2 sin α, –w/2 sin α – h/2 cos α)
The TL corner is at (–w/2 cos α – h/2 sin α, –w/2 sin α + h/2 cos α)

The slope (BL to TL) is

$$\frac{-w/2 \sin \alpha + h/2 \cos \alpha - (-w/2 \sin \alpha - h/2 \cos \alpha)}{-w/2 \cos \alpha - h/2 \sin \alpha - (-w/2 \cos \alpha + h/2 \sin \alpha)}$$

or

$$\frac{-w/2 \sin \alpha + h/2 \cos \alpha + w/2 \sin \alpha + h/2 \cos \alpha}{-w/2 \cos \alpha - h/2 \sin \alpha + w/2 \cos \alpha - h/2 \sin \alpha}$$

or

$$\frac{h \cos \alpha}{-h \sin \alpha}$$

or

$$-1/\tan \alpha$$

In y = mx +b solve for b, by substituting the BL corner in it.

$$y = -x/\tan \alpha + b$$

or

$$b = x/\tan \alpha + y$$

or

$$b = (-w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha + (-w/2 \sin \alpha - h/2 \cos \alpha)$$

or

$$b = (-w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

so

$$y = -x/\tan \alpha + (-w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

so the equation for the Left side is

$$y = (-x - w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

Calculating and plotting shows this to be correct.

Bottom rotated side

The BL corner is at $(-w/2 \cos \alpha + h/2 \sin \alpha, -w/2 \sin \alpha - h/2 \cos \alpha)$
The BR corner is $(w/2 \cos \alpha + h/2 \sin \alpha, w/2 \sin \alpha - h/2 \cos \alpha)$

The slope (from BL to BR) is

$$\frac{w/2 \sin \alpha - h/2 \cos \alpha - (-w/2 \sin \alpha - h/2 \cos \alpha)}{w/2 \cos \alpha + h/2 \sin \alpha - (-w/2 \cos \alpha + h/2 \sin \alpha)}$$

or

$$\frac{w/2 \sin \alpha - h/2 \cos \alpha + w/2 \sin \alpha + h/2 \cos \alpha)}{w/2 \cos \alpha + h/2 \sin \alpha + w/2 \cos \alpha - h/2 \sin \alpha)}$$

or

$$\frac{w \sin \alpha}{w \cos \alpha}$$

or

$$\tan \alpha$$

In y = mx +b solve for b, by substituting the BL corner in it.

$$y = mx + b$$

$$y = x \tan \alpha + b$$

Given BL, solve for b

$$b = -x \tan \alpha + y$$

or

$$b = -(-w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha + (-w/2 \sin \alpha - h/2 \cos \alpha)$$

or

$$b = (w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

so the equation for the Bottom side is

$$y = x \tan \alpha + (w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

or

$$y = (x + w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

Calculating and plotting shows this to be correct.

Determine points where sides of the rotated image intercept the sides of the frame

For **anti-clockwise** rotation of the image, the following intercept points are of interest:

From the following sections, the intercept points are:

Side of frame – side of image	Formulae for coordinates	In test case
right – bottom	[w/2, (w/2 + w/2 cos α – h/2 sin α) tan α –w/2 sin α – h/2 cos α]	[10,–4.3]
bottom – left	[(h/2 – w/2 sin α – h/2 cos α)tan α –w/2 cos α + h/2 sin α, –h/2)	[–9.1,–6]

These assume anticlockwise rotation. They can also be used for clockwise rotation, due to symmetry.

Right side of frame with rotated bottom of image

The equation for the rotated bottom of image is

$$y = (x + w/2 \cos \alpha – h/2 \sin \alpha)\ \tan \alpha –w/2 \sin \alpha – h/2 \cos \alpha$$

The equation for the right side of the frame is $x = w/2$

So the intercept is

$$y = (w/2 + w/2 \cos \alpha – h/2 \sin \alpha)\ \tan \alpha – w/2 \sin \alpha – h/2 \cos \alpha$$

Calculating and plotting shows this to be correct.

Bottom of frame with rotated left side of image

The equation for the rotated left side of image is

$$y = (–x – w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha – w/2 \sin \alpha – h/2 \cos \alpha$$

The equation for the bottom of the frame is $y = –h/2$

So: solve for x:

$$–h/2 = (–x – w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha – w/2 \sin \alpha – h/2 \cos \alpha$$

or

$$–h/2 = –x/\tan \alpha + (–w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha – w/2 \sin \alpha – h/2 \cos \alpha$$

or

$$x/\tan \alpha = h/2 + (–w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha – w/2 \sin \alpha – h/2 \cos \alpha$$

or

$$x/\tan \alpha = (–w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha + h/2 – w/2 \sin \alpha – h/2 \cos \alpha$$

or

$$x = (h/2 – w/2 \sin \alpha – h/2 \cos \alpha)\tan \alpha –w/2 \cos \alpha + h/2 \sin \alpha$$

Calculating and plotting shows $x = –9.09$; this is correct

Cropping for anti-clockwise rotation of the image

The diagram below suggest that good cropping is achieved by cropping from all sides:

- in the x direction: by minus intercept of bottom of frame and left side of rotated image, plus w/2
- in the y direction: by intercept of right side of frame and bottom of rotated image, plus h/2

From above, the x intercept of bottom of frame and left side of rotated image (piBL) is

$$(h/2 – w/2 \sin \alpha – h/2 \cos \alpha)\tan \alpha –w/2 \cos \alpha + h/2 \sin \alpha$$

and the y intercept of right side of frame and bottom of rotated image (piRB) is

$$(w/2 + w/2 \cos \alpha – h/2 \sin \alpha) \tan \alpha –w/2 \sin \alpha – h/2 \cos \alpha$$

In the test case this is:

- in the x direction: 9.1
- in the y direction: –4.3

Consider `_GDIPlus_GraphicsDrawImageRectRect($hGraphics, $hImage, $nSrcX, $nSrcY, $nSrcWidth, ...)`

nsrcX = Ibl + w/2

or

nsrcX = (h/2 – w/2 sin α – h/2 cos α)tan α –w/2 cos α + h/2 sin α + w/2

AutoIt:

```
$nsrcX = ($h/2 - $w/2*sin($a) - $h/2*cos($a))*tan($a) - $w/2*cos($a) + $h/2*sin($a) + $w/2
```

and

nsrcY = Ibl + h/2

or

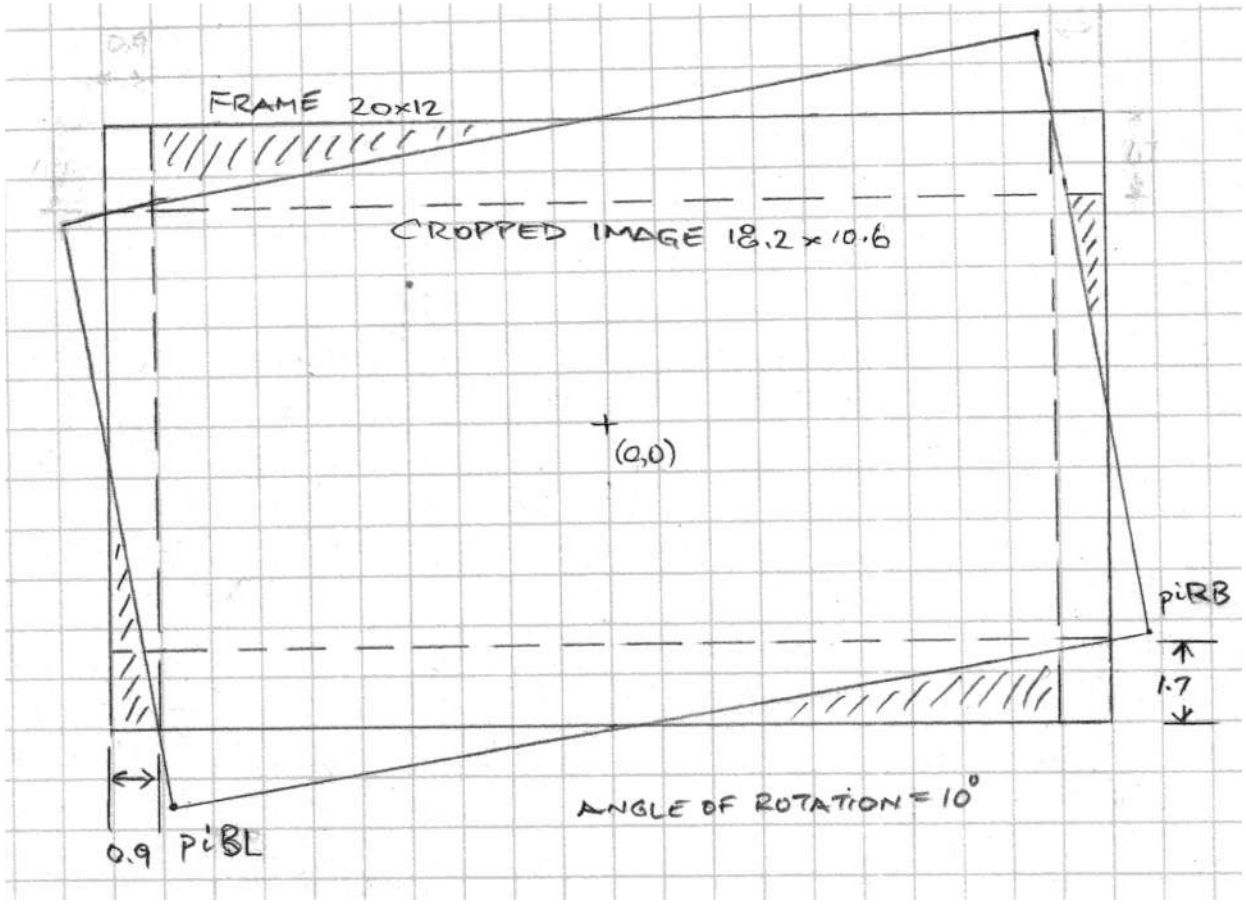
nsrcY = (w/2 + w/2 cos α – h/2 sin α) tan α – w/2 sin α – h/2 cos α + h/2

AutoIt:

```
$nsrcY = ($w/2 + $w/2*cos($a) - $h/2*sin($a))*tan($a) - w/2*sin($a) - $h/2*cos($a) + $h/2
```

For the test case, the source parameters are:

`$nSrcX`	0.9
`$nSrcY`	1.7
`$nSrcWidth`	20 – 2(0.9) = 18.2
`$nSrcHeight`	12 – 2(1.7) = 10.6



This cropping is good for small rotation angles, but is not the minimum. For w:h = 4:3 (typical for a photograph) and 3 degree rotation, this method crops to 89% of the original photograph..

This also appears to apply to clockwise rotation of the image.

Limit on rotation angle

This occurs when the intercept of the left side of the frame and the top of the rotated top of the image reaches 0.

The formula for the x coordinate of this intercept is $(-w/2 + w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha$. The limit is when this formula evaluates to zero.

An AutoIt script shows, for the test case, that this occurs at 37 degrees. For this case, asin(h/w) = 37 degrees.

I do not know of a way of solving for α algebraically.

As the rotation angle is increased, the cropped image rapidly becomes smaller, so auto-cropping an image rotated by more than 10 degrees does not produce a useful result.

<u>Dragging the horizon line when it is off horizontal</u>

The ends of the line are (x0,y0) and (x1,y1.

The general equation is y = mx + b

$$m = \frac{y1 - y0}{x1 - x0}$$

```
b = y - mx
```

$$b = y - \frac{y1 - y0}{x1 - x0} x$$

Substituting in (x0,y0

```
b = y0 - (y1 - y0)/(x1-x0)*x0
```

So

```
y = (y1 - y0)/(x1 - x0)*x + y0 - (y1 - y0)/(x1 - x0) + y0
```

or

```
y = (y1 - y0)/(x1 - x0)*(x - x0) + y0
```

When a user starts to drag the line, x' is known.

So y' can be determined. Call `MoveMove()` to move the cursor to this position before dragging in a while loop.

<u>Grabbing the line with the mouse</u>

An earlier version of the script called `_GDIPlus_PathIsOutlineVisiblePoint()` to determine when the user mouse-downs on the line (or on an end of the line). The problem with this method was that extreme precision was required when downing the mouse; selecting the line (or ends of it) failed if the mouse wasn't exactly on the line. This was alleviated by placing a marker on each end of the line, large enough for the user to click in.

I decided to take different approach. I surrounded the line with an invisible polygon (a "fence"), inserted this figure into a Path, and called `_GDIPlus_PathIsVisiblePoint()` to do the determination of when the user had

clicked near the line. The pen for this Path is transparent, so to the user there appears to be only a line (without the polygon). Per the Help, this function works on filled figures.

Appendix

The appendix is here so work that it turned out was not needed is preserved. It may be useful to others.

Determine the equations for other sides of the rotated image

Right rotated side

The TR corner is at $(w/2 \cos \alpha - h/2 \sin \alpha, w/2 \sin \alpha + h/2 \cos \alpha)$
The BR corner is at $(w/2 \cos \alpha + h/2 \sin \alpha, w/2 \sin \alpha - h/2 \cos \alpha)$

The slope (LR to TR) is

$$\frac{w/2 \sin \alpha + h/2 \cos \alpha - (w/2 \sin \alpha - h/2 \cos \alpha)}{w/2 \cos \alpha - h/2 \sin \alpha - (w/2 \cos \alpha + h/2 \sin \alpha)}$$

or

$$\frac{w/2 \sin \alpha + h/2 \cos \alpha - w/2 \sin \alpha + h/2 \cos \alpha}{w/2 \cos \alpha - h/2 \sin \alpha - w/2 \cos \alpha - h/2 \sin \alpha}$$

or

$$\frac{h \cos \alpha}{- h \sin \alpha}$$

or

$$-1/\tan \alpha$$

This is the same as for the Left side, so we have

$$b = x/\tan \alpha + y$$

Substituting in the values for BR we have

$$b = (w/2 \cos \alpha + h/2 \sin \alpha) /\tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha$$

so the equation for the Right side is

$$y = -x/\tan \alpha + (w/2 \cos \alpha + h/2 \sin \alpha) /\tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha$$

or

$$y = (- x + w/2 \cos \alpha + h/2 \sin \alpha) /\tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha$$

Calculating and plotting shows this to be correct.

Top rotated side

The TL corner is at $(-w/2 \cos \alpha - h/2 \sin \alpha, -w/2 \sin \alpha + h/2 \cos \alpha)$
The TR corner is $(w/2 \cos \alpha - h/2 \sin \alpha, w/2 \sin \alpha + h/2 \cos \alpha)$

The slope (from TL to TR) is

$$\frac{w/2 \sin \alpha + h/2 \cos \alpha - (-w/2 \sin \alpha + h/2 \cos \alpha)}{w/2 \cos \alpha - h/2 \sin \alpha - (-w/2 \cos \alpha - h/2 \sin \alpha)}$$

or

$$\frac{w/2 \sin \alpha + h/2 \cos \alpha + w/2 \sin \alpha - h/2 \cos \alpha)}{w/2 \cos \alpha - h/2 \sin \alpha + w/2 \cos \alpha + h/2 \sin \alpha)}$$

or

$$\frac{w \sin \alpha}{w \cos \alpha}$$

or

$$\tan \alpha$$

In y = mx + b solve for b, by substituting the TL corner in it.

$$b = -mx + y$$

or

$$b = -x \tan \alpha + y$$

or

$$b = -(-w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha + (-w/2 \sin \alpha + h/2 \cos \alpha)$$

or

$$b = (w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha)$$

so the equation for the Top side is

$$y = x \tan \alpha + b$$

or

$$y = x \tan \alpha + (w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha)$$

$$y = (x + w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha)$$

Calculating and plotting shows this to be correct.

Determine other points where sides of the rotated image intercept the sides of the frame

From the following sections, the intercept points are:

Side of frame – side of image	Formulae for coordinates	In test case
left – left	$[-w/2, (w/2 - w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha]$	[–10,–0.9]
left – top	$[-w/2, (-w/2 + w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha]$	[–10,4.3]
right – right	$[w/2, (-w/2 + w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha]$	[10,0.9]
bottom – bottom	$[-w/2 \cos \alpha + h/2 \sin \alpha + (w/2 \sin \alpha + h/2 \cos \alpha - h/2)/ \tan \alpha, -h/2)$	[0.5,–6]
top – right	$[(w/2 \sin \alpha - h/2 \cos \alpha - h/2)/\tan \alpha + w/2 \cos \alpha + h/2 \sin \alpha, h/2]$	[9.1,6]
top – top	$[(w/2 \sin \alpha - h/2 \cos \alpha + h/2)/\tan \alpha - w/2 \cos \alpha - h/2 \sin \alpha, h/2]$	[–1.6,6]

Left side of frame with rotated left side of image

The equation for the rotated left side of image is

$$y = (-x - w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

The equation for the left side of the frame is $x = -w/2$

So the intercept is

$$y = (w/2 - w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

Calculating and plotting shows this to be correct.

Left side of frame with rotated top of image

The equation for the rotated top of image is

$$y = (x + w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha$$

The equation for the left side of the frame is $x = -w/2$

So the intercept is

$$y = (-w/2 + w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha$$

Calculating and plotting shows this to be correct.

Right side of frame with rotated right side of image

The equation for the rotated right side of image is

$$y = (- x + w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha$$

The equation for the right side of the frame is $x = w/2$

So the intercept is

$$y = (-w/2 + w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

Calculating and plotting shows this to be correct.

Bottom of frame with rotated bottom of image

The equation for the rotated bottom of image is

$$y = (x + w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

The equation for the bottom of the frame is $y = -h/2$

So solve for x:

$$-h/2 = (x + w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

or

$$-h/2 = x \tan \alpha + (w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha - h/2 \cos \alpha$$

or

$$x \tan \alpha = -(w/2 \cos \alpha - h/2 \sin \alpha) \tan \alpha + w/2 \sin \alpha + h/2 \cos \alpha - h/2$$

or

$$x = -w/2 \cos \alpha + h/2 \sin \alpha + (w/2 \sin \alpha + h/2 \cos \alpha - h/2)/ \tan \alpha$$

Calculating and plotting shows $x = 0.53$; this is correct

Top of frame with rotated right side of image

The equation for the rotated right side of image is

$$y = (- x + w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha$$

The equation for the top of the frame is $y = h/2$

So solve for x:

$$h/2 = (- x + w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha$$

or

$$h/2 = -x /\tan \alpha + (w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha$$

or

$$x /\tan \alpha = (w/2 \cos \alpha + h/2 \sin \alpha)/\tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha - h/2$$

or

$$x = (w/2 \sin \alpha - h/2 \cos \alpha - h/2)/\tan \alpha + w/2 \cos \alpha + h/2 \sin \alpha$$

Calculating and plotting shows x = 9.10; this is correct

Top of frame with rotated top of image

The equation for the rotated top of image is

$$y = (x + w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha$$

The equation for the top of the frame is $y = h/2$

So solve for x:

$$h/2 = (x + w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha$$

$$h/2 = x \tan \alpha + (w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha$$

$$h/2 - x \tan \alpha = (w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha$$

$$- x \tan \alpha = (w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha - w/2 \sin \alpha + h/2 \cos \alpha - h/2$$

$$x \tan \alpha = - (w/2 \cos \alpha + h/2 \sin \alpha) \tan \alpha + w/2 \sin \alpha - h/2 \cos \alpha + h/2$$

$$x = - (w/2 \cos \alpha + h/2 \sin \alpha) + (w/2 \sin \alpha - h/2 \cos \alpha + h/2)/ \tan \alpha$$

$$x = (w/2 \sin \alpha - h/2 \cos \alpha + h/2)/ \tan \alpha - w/2 \cos \alpha - h/2 \sin \alpha$$

Calculating and plotting shows x =–1.6; this is correct